Basic income and unemployment in a unionized economy

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Abstract

This paper develops a dynamic general equilibrium model of a unionized economy to analyze the impact of unconditional basic income schemes on unemployment. Starting from a given level of the unemployment benefits, two reforms are envisaged : one where these benefits are replaced by a higher unconditional grant (the full basic income) and another where the income of the unemployed remains unchanged (the partial basic income). Assuming a proportional tax on earnings and a balanced budget of the State, it is shown that the equilibrium unemployment rate decreases if a partial basic income is implemented. The same conclusion holds for a sufficiently small full basic income.

Keywords: Basic income; unconditional grant; wage bargaining; unemployment

JEL classification : H2, H3, J5.

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1 Introduction

A basic or unconditional income consists of the payment of a grant to every adult citizen. This idea has taken various forms and has received various names : *social dividend* (Lange, 1936, and Meade, 1989) and *basic income* (Parker, 1989, and Atkinson, 1995). The ethical foundations of such a proposal have been developed at length (see, e.g., Van Parijs 1992, Van Parijs, 1995). Yet, economists are often worried by the economic implications of such a scheme. In the European context of high and persistent unemployment, it is in particular justified to raise the question : What is the implication of a basic income on the level of unemployment? It has been argued that the combination of a basic income and a deregulated labor market could reconcile labor market efficiency and income protection (Standing, 1992). Whether a competitive outcome would emerge from a deregulated labor market is an open question not raised in this paper. I assume the existence of unions with a given bargaining power and analyze how the implementation of a basic income affects the level of unemployment.

Two variants of the basic income proposal are considered in this paper : The pure form of the scheme and a partial basic income. In the first case, henceforth called *a full basic income scheme*, the unconditional grant replaces all social insurance and social assistance benefits for the population of working age. If the basic income has to be financed by the income tax and if, for whatever reason, the highest marginal tax rates cannot be raised, the initial and intermediate tax rates have to increase substantially in order to finance a reasonable basic income level. The political opposition to such a reform and uncertainties about the behavior of economic agents when tax parameters are deeply modified have led to proposals for *a partial basic income* (Parker, 1989). The unconditional grant is smaller than in the first case. The social insurance and social assistance benefits that are lower than the basic income disappear and the others are in a way or another reduced by the amount of the partial basic income.

These two variants of the basic income proposal will be analyzed in a dynamic model of collective bargaining developed by Manning (1991, 1993) and Cahuc and Zylberberg (1996). The analysis will be developed in a general equilibrium setting with two factors, labor and capital. The State levies a proportional tax on earnings and his budget has to be balanced in each period. In this setting, imperfect competition on the labor market causes unemployment. It is assumed that the unions' power and preferences and the unemployment benefit system are exogenously given. The paper raises the question whether the two variants of the basic income scheme are a way of fighting unemployment or not. This analysis does not have the ambition to provide a definite answer to this question. It nevertheless lead to rather clear-cut conclusions which should be checked in a richer framework with heterogeneous workers and/or sectors, non linear taxation and the like.

The conclusions of this paper are as follows : (1) Compared to the case with an unemployment insurance system but without basic income, the equilibrium unemployment rate is always lower if a partial basic income scheme is implemented; the same holds if the unemployment insurance system is abolished and replaced by a full basic income scheme provided that the latter is sufficiently small; (2) the equilibrium unemployment rate is a decreasing function of the level of the partial basic income; (3) the equilibrium unemployment rate is an increasing function of the level of the full basic income if workers are risk averse; (4) the equilibrium tax wedge increases with the level of the basic income schemes while the net wage rate decreases; (5) introducing a sufficiently small partial basic income can be a Pareto improvement.

The literature on the macroeconomic effects of a basic income is not much developed so far. As far as I know, the literature is only concerned with the so-called full basic income scheme. Bowles (1992) focuses on the relationship between a universal grant and work effort. He concludes that a small unconditional grant can be introduced without reducing the pre-grant level of profits. The main mechanism behind this result is that the grant lowers the workers' fall-back position. Atkinson (1995) analyses the switch from unemployment benefits financed by a payroll tax to a basic income scheme and a flat income tax. He shows that this reform reduces the unemployment and wage levels in a dual labor market. Yet, this conclusion appears to depend on the institutional features of the unemployment benefit system. In a fix-price model, Késenne (1991) explains that a basic income helps to clear the labor market through a reduction of labor supply. Késenne (1993) develops a static macro model where output prices are exogenously fixed and wages are the outcome of an efficient bargain. His results emphasize the roles of labor supply responses and of fall-back positions in the bargaining process.

This paper is organized as follows. To provide insight into the main effects of a basic income on wage bargaining, section 2 proposes a simple partial equilibrium and static model. Section 3 presents a dynamic general equilibrium model of a unionized economy. Starting from the latter model, section 4 develops the main comparative static results about the effect of a basic income on unemployment. Section 5 presents a numerical example and section 6 concludes the paper.

2 A static model in partial equilibrium

Assume a firm with a quasi-Cobb-Douglas technology $(AL)^\alpha$ where L denotes the homogeneous labor factor, A is a positive parameter and $0 < \alpha < 1$. Assume also perfect competition on the goods market. Let w be the *net* real wage and τ the exogenous tax wedge. The labor demand and the profit function of this firm are defined as

$$\begin{aligned} L(w,\tau) &= \frac{1}{A}\left(\frac{w(1+\tau)}{A\alpha}\right)^{\frac{1}{\alpha-1}} \quad (1)\\ \pi(w,\tau) &= \alpha^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w(1+\tau)}{A}\right)^{\frac{\alpha}{\alpha-1}}. \quad (2)\end{aligned}$$

Workers are endowed with a utility function $v(R)$, where R denotes net real income, $v'(R) > 0$ and $v''(R) \leq 0$. The working time is exogenous and normalized to one. The utility of an employed worker is $v(w + B)$, where B is the unconditional basic income $(B \geq 0)$. The utility of an unemployed, v_u, is different depending on whether the full or the partial basic income scheme applies. The partial basic income is such that $v_u = v(Z)$, where Z is the level of real unemployment benefits, for Z is reduced by the amount of the partial basic income, while the unemployed, as any other adult citizen, receives the partial basic income. So, his net real income is Z. The full basic income scheme replaces unemployment benefits, so that now $v_u = v(B)$. To facilitate the comparison between these two variants, it is assumed that the level of Z is exogenously given ($Z > 0$). Moreover a full basic income scheme is by assumption such that B is higher than Z. To sum up, in this analysis, the utility of an unemployed is defined as

$$v_u = \begin{cases} v(Z) & \text{if } Z > B > 0 \text{ (partial basic income scheme)} \\ v(B) & \text{if } B \geq Z > 0 \text{ (full basic income scheme).} \end{cases} \tag{3}$$

These assumptions offer a clear distinction between the two variants. Yet, they preclude any evaluation of the case where the unemployment benefit system is abolished and replaced by a *full* but low basic income scheme (i.e. $Z > B > 0$ but $v_u = v(B)$). Moreover, one could perfectly well imagine that a *full* basic income be implemented simultaneously with an unemployment insurance scheme. The pros and cons of such a combination are not discussed in this paper.

Following Manning (1991, 1993), assume that the union adopts the following objective function :

$$\mathcal{V} = L^{\psi}(v(w + B) - v_g), \tag{4}$$

where ψ is a nonnegative parameter representing "union's preferences for employment relative to wages" (Manning, 1993, p.101). It is typically assumed that $\psi \in [0, 1]$. v_g is the outside option or more precisely "some measure of the value of alternatives available elsewhere in the economy to workers who lose their jobs in this firm" (Manning, 1993, p. 100-101). Hence, in this static partial equilibrium setting, let $v_g = v_u$ defined above.

Assume that the firm decides upon the employment level (the *right-to-manage* assumption) and that w is set to maximize a Nash product $(\pi - \pi_0)^{1-\gamma}(\mathcal{V} - \mathcal{V}_0)^{\gamma}$ where γ is the so-called bargaining power of the union ($\gamma \in [0, 1]$). The fall-back positions of the union and the firm $(\pi_0, \mathcal{V}_0)$ are assumed to be zero[1]. Remembering the labor demand and profit functions (1)-(2) and ignoring constant terms, the Nash product can be written as

$$\max_w w^{\left(\frac{\alpha(1-\gamma)}{\alpha-1}+\frac{\psi\gamma}{\alpha-1}\right)}(v(w+B)-v_u)^{\gamma}. \tag{5}$$

Assume the usual hypothesis according to which the level of the unemployment benefit is independent of the negotiated wage rate in a *particular* firm. Extend this assumption to the basic income. The first-order condition of problem (5) can then be formulated as follows :

$$\frac{v(w+B)-v_u}{wv'(w+B)} = \mu, \text{ where } \mu = \frac{\gamma(1-\alpha)}{\alpha(1-\gamma)+\psi\gamma} > 0. \tag{6}$$

The second-order condition is satisfied if $\mu < 1$ or, equivalently, $\gamma(1-\psi) < \alpha$. This is always true if the union has a utilitarian objective ($\psi = 1$). In a seniority model ($\psi = 0$), the condition $\mu < 1$ imposes an upper-bound on the union's bargaining power. Henceforth, the condition $\mu < 1$ is assumed to hold. μ is smaller the higher the absolute values of the elasticities of profits and labor demand with respect to the real wage rate (respectively, $\frac{\alpha}{1-\alpha}$ and $\frac{1}{1-\alpha}$). μ is also smaller the higher the union's relative preference for employment, ψ.

The analysis of the effect of the basic income on employment is straightforward in this partial equilibrium setting. If an increase in B lowers the wage rate w, labor demand increases and so does employment (assuming no labor supply constraint). Therefore the analysis will now focus on the sign of $\frac{\partial w}{\partial B}$. Let us start with the partial basic income policy (i.e. $v_u = v(Z)$). In equation (6), as B increases, $v(w+B)-v_u$ increases too for any value of w. In addition, $v'(w+B)$ decreases if workers are risk averse or stays constant if they are risk neutral. Hence, the ratio on the left-hand side of equality (6) increases. To keep it equal to the constant μ (i.e. to maintain optimality), w has to be lower. More precisely,

$$\frac{\partial w}{\partial B} = -1 - \frac{\mu v'(w+B)}{(1-\mu)v'(w+B) - \mu v''(w+B)} < -1. \tag{7}$$

[1]As in Manning (1991), this means that the inside and the outside options are assumed to be equal.

This partial equilibrium result implies that introducing a partial basic income or increasing it has a favorable impact on the employment level.

The intuition behind this conclusion is easily grasped. Equation (5) implies that an increase in the *partial* basic income compensated by an identical decrease in the net wage does not change the rent $v(w+B)-v_u$ of a union member. Yet, the reduction in wage will enhance labor demand and profits. So, a more than proportional wage cut can typically be expected. Notice that in this partial equilibrium setting there is no difference between a partial basic income and a wage subsidy paid to each employee.

Let us now turn to the *full* basic income policy. The optimal condition (6) applies here too if v_u is now interpreted as $v(B)$. If workers are risk neutral, $v(w+B)-v(B)$ stays constant whatever the value of B. Put differently, the wage rate is not modified by the presence of a basic income. However, if workers are risk averse, the numerator of (6) decreases with B. So does the denominator. Hence, the net effect is in general ambiguous. It can be checked that $\frac{\partial w}{\partial B} > 0$ (resp. < 0) if

$$-(w+B)\frac{v''(w+B)}{v'(w+B)} < \text{ (resp. } >) \, \frac{1}{\mu}\left(\frac{v'(B)}{v'(w+B)} - 1\right)\left(1+\frac{B}{w}\right) > 0. \tag{8}$$

Put another way, in a partial equilibrium setting, a full basic income can favor wage moderation only if workers are sufficiently risk averse.

3 A dynamic model in general equilibrium

This section intends to verify whether the previous results carry over in a more comprehensive model of a unionized economy. A dynamic general equilibrium model with identical agents is developed. This model draws upon chapter 8 of Cahuc and Zylberberg (1996), henceforth *CZ*. The model of *CZ* itself is inspired by Manning (1991,1993) and Layard and Nickell (1990). There is implicitly an international financial market with perfect mobility. The economy is assumed to be a small one which faces an exogenous interest rate r. The basic income and unemployment benefits are financed by a proportional tax on earnings. The budget of the State is assumed to balance in each period. I keep the hypothesis according to which the levels of the unemployment benefit and the basic income are independent of the negotiated wage rate in a *particular* firm.

3.1 Assumptions and notations

Assume a deterministic setting with, in each period t, n identical firms, N homogeneous workers and M inactive individuals entitled to a basic income. Each of the n firm owners bargains over wages with a firm-specific union. The former decides unilaterally on employment and on the level of investment. Firms and workers are infinitely lived agents with perfect foresight. In a given period t, the sequence of decisions is as follows :

1. At the beginning of the period, a bargaining over the current wage level takes place in each firm (wages are only set for one period). If an agreement is reached, the employees (whose number is defined at stage 2) receives each a *net* real wage w_t at the end of period t.

2. In firms where there is a collective agreement, the firm determines labor demand and the investment level for the current period. Given w_t, the employment level is fixed by labor demand and production occurs. Without an agreement at stage 1, nothing is produced during the current period. Yet, the firm will have the opportunity to bargain and to hire workers (without hiring costs) in $t + 1$.

3. The tax wedge is adjusted for the current period in order to balance the public budget constraint.

4. At the end of the period, an exogenous fraction q of the employees leaves the firm and enters unemployment[2].

[2]The following comment of Layard and Nickell (1990) (p.780) applies here, too :

> In reality, of course, turnover is also generated by exogenous demand shocks of various kinds. This is ignored here, since we do not wish to introduce such explicit stochastic elements into the model. However, we feel that our model will mimic closely the consequences of a stochastic (...) model.

Firms

To simplify the exposition, the n firms produce a homogeneous good and sell it on a competitive market. The output price is normalized to 1. All firms are assumed to be identical. To save on notation, no subscript will be added to identify a particular firm. When it determines labor demand and the investment level, each firm takes as given the (constant) interest rate r and the (constant) depreciation rate δ ($\delta \in [0, 1]$). At this stage, each firm takes also the real wage cost, $w_t(1 + \tau_t)$, as given (by assumption, a particular firm is small and cannot influence the tax wedge τ_t).

In period t, given the initial capital stock K_t, each firm chooses its investment level I_t, such that $K_{t+1} = (1 - \delta)K_t + I_t$. Let L_t denote labor demanded by the firm. Assume a Cobb-Douglas technology with constant returns to scale : $(AL_t)^\alpha K_t^{1-\alpha}, A > 0$. In period $t = 0$, given K_0, the firm maximizes the following objective function :

$$\max_{\{L_t, K_{t+1}\}_{t \geq 0}} \sum_{t=0}^{\infty} (\frac{1}{1+r})^t [(AL_t)^\alpha K_t^{1-\alpha} - w_t(1+\tau_t)L_t - (K_{t+1} - (1-\delta)K_t)]. \tag{9}$$

The first-order conditions of problem (9) can be written as

$$\alpha A \left(\frac{K_t}{AL_t}\right)^{1-\alpha} = w_t(1+\tau_t), \ \forall t \geq 0 \tag{10}$$

$$(1-\alpha)\left(\frac{K_{t+1}}{AL_{t+1}}\right)^{-\alpha} = \delta + r, \ \forall t \geq 0. \tag{11}$$

Given K_t, equation (10) determines L_t as a function of w_t and τ_t. The capital stock K_{t+1}, and the investment level I_t, are derived from equation (11), as a function of the next period employment level L_{t+1}.

In $t = 0$, the capital stock is fixed and equation (10) defines a downward-sloping labor demand curve. For $t \geq 1$, given the constant returns to scale assumption, the input demand functions are not well defined by equations (10) and (11). Only the capital-labor ratio $\frac{K_t}{L_t}$ is defined. Moreover equations (10) and (11) evaluated respectively at time t and $t - 1$ determine the wage cost at time t as a function of the structural parameters of problem (9). This relationship is actually a real input prices frontier[3]. It writes :

$$(1+\tau_t)w_t = C, \text{ where } C = \alpha A \left(\frac{\delta + r}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} > 0. \tag{12}$$

This equation means that the wage cost is exogenously fixed by the structural parameters characterizing the firm and the economy (e.g., the interest rate r). A similar property would have hold if we had assumed imperfectly competitive firms facing an isoelastic demand curve for their output (see *CZ*, p. 487-490). Yet, the perfect competition hypothesis implies that the level of output supplied by the firm is here indeterminate. Therefore, the bargaining over wages described below will actually determine the unemployment level, from which the output level can be derived.

Let π_t be current profits net of investment, i.e. $\pi_t = \max_{L_t}((AL_t)^\alpha K_t^{1-\alpha} - w_t(1+\tau_t)L_t)$. With a Cobb-Douglas technology, one has

$$\pi_t = \alpha^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) K_t \left(\frac{w_t(1+\tau_t)}{A}\right)^{\frac{\alpha}{\alpha-1}}. \tag{13}$$

Discounted profits at time t, Π_t, are defined as

$$\Pi_t = \pi_t - I_t + \beta \Pi_{t+1}. \tag{14}$$

where $\beta = \frac{1}{1+r}$ is the discount factor ($\beta \in]0,1[$).

Workers

Each of the N homogeneous workers supplies one unit of labor. His instantaneous utility function is $v(R_t), v' > 0, v'' < 0$, where R_t denotes net real income in period t (by assumption, there is no saving). Notice that with such a specification neither the unemployment benefit nor the basic income can influence labor supply. Hence, the size of the labor force will be exogenous.

At the end of period t, each employee leaves the firm with an exogenous probability q, $q \in]0,1[$. He is then unemployed at the beginning of period $t+1$ and will be hired by a

[3] The constant returns to scale assumption implies that the long-run marginal production cost is constant. The firm is ready to supply any amount of output provided that the output price is equal to the marginal cost. This equality can be rewritten as a real input prices frontier.

firm with probability a_{t+1} [4]. Hence, in period t, the intertemporal discounted utility of a job in the firm, V_e^t, is given by the following expression :

$$V_e^t = v(w_t + B_t) \ + \ \beta\{q[a_{t+1}\overline{V_e^{t+1}} + (1-a_{t+1})V_u^{t+1}] \ + \ (1-q)V_e^{t+1}\}, \tag{15}$$

where B_t denotes the level of the basic income at time t, $\overline{V_e^{t+1}}$ is the intertemporal discounted utility of a job on average in the economy in period $t+1$ and V_u^{t+1} is the intertemporal discounted utility of being unemployed in $t+1$. $\overline{V_e^{t+1}}$ is of the same form as (15) with only one difference : The average net real wage in the economy, $\overline{w_t}$, replaces w_t.

The intertemporal discounted utility of being unemployed at time t, V_u^t, is given by

$$V_u^t = v_u^t + \beta\{a_{t+1}\overline{V_e^{t+1}} + (1-a_{t+1})V_u^{t+1}\}, \tag{16}$$

where $v_u^t = v(Z_t)$ with a partial basic income and $v_u^t = v(B_t)$ with a full basic income.

Wage-setting

Following Manning (1991, 1993), assume that the firm-specific union adopts the following objective function :

$$L_t^{\psi}(V_e^t - V_g^t), \tag{17}$$

where ψ has the same meaning as in section 2. Redundant workers are assumed to be immediately rehired in another firm with probability a_t. Hence,

$$V_g^t = a_t\overline{V_e^t} + (1-a_t)V_u^t. \tag{18}$$

The objective function (17) is fairly general. It should however be noticed that it simplifies drastically the analysis of the intertemporal model, for the time-dependent size of the union does not appear in (17) (compare, e.g., with Huizinga and Schiantarelli, 1992).

Assume that the current real wage w_t is set to maximize a Nash product. As in section 2, the fall-back position of the union is assumed to be zero. As argued by *CZ* (p. 470-471), in the absence of an agreement, nothing is produced but future profits and,

[4]Workers move freely from one firm to another.

hence, investment are not affected. It has indeed be assumed that the firm will have the opportunity to bargain and to hire workers (without hiring costs) in $t+1$. Hence, the firm's component in the Nash product, i.e., the difference between intertemporal discounted profits in case of an agreement, Π_t, and in the absence of an agreement, $-I_t + \beta\Pi_{t+1}$, is simply π_t. Remembering (13) and ignoring constant and predetermined terms, the Nash program writes

$$\max_{w_t}(w_t)^{\frac{\alpha(1-\gamma)}{\alpha-1}} L_t^{\psi\gamma}(V_e^t - V_g^t)^{\gamma}, \tag{19}$$

where γ is the so-called bargaining power of the union. The tax wedge τ_t does not appear in (19) because the players take it as given. The first-order condition of this problem can be written as

$$V_e^t - V_g^t = \mu w_t v'(w_t + B_t), \tag{20}$$

where μ has been defined in equation (6). The second-order condition is satisfied if $\mu < 1$ (about this requirement, see section 2).

3.2 The equilibrium

In the initial period ($t = 0$), this economy is characterized by an aggregate labor demand curve (derived from (10)), the wage-setting equation (20) and by the budget constraint of the State. For $t \geq 1$, this economy is fully characterized by the real input prices frontier (12) and by the same wage-setting equation and balanced budget constraint.

The wage-setting equations

Since all firms and unions' characteristics are identical, at an equilibrium, $w_t = \overline{w_t}$ and $V_e^t = \overline{V_e^t}$. Then (18) implies that

$$V_e^t - V_g^t = (1 - a_t)(V_e^t - V_u^t). \tag{21}$$

From (15) and (16), it can be seen that

$$V_e^t - V_u^t \ = \ v(w_t + B_t) - v_u^t + \beta(1 - q)(V_e^{t+1} - V_g^{t+1}). \tag{22}$$

Combining (20), (21) and (22) leads to the following expression :

$$\frac{v(w_t + B_t) - v_u^t}{\mu w_t v'(w_t + B_t)} + \beta(1-q)\frac{w_{t+1}}{w_t}\frac{v'(w_{t+1} + B_{t+1})}{v'(w_t + B_t)} = \frac{1}{1-a_t}. \tag{23}$$

In this equality, the first component of the left-hand side has already been met in the static model of section 2. In equation (6), its optimal level was equal to one. This is no more the case in the dynamic setting essentially because of the separation rate q.

The right-hand-side of (23) can be rewritten as a function of the current and previous unemployment rate. For the current unemployment level is made of those who where unemployed at the beginning of this period and who are not currently hired. The former group is made of those who were unemployed in period $t-1$ and those who entered unemployment at the end of the same period. After division by the size of the labor force, N, this definition writes

$$u_t = (1-a_t)(q + (1-q)u_{t-1}), \tag{24}$$

where u_t is the unemployment rate in period t. Substituting $(1-a_t)$ from this equation into (23) leads to an implicit wage equation where the current net real wage rate is a function of the current and past unemployment rate, the anticipated wage rate in $t+1$ and the current and anticipated levels of the allowances (B and, where appropriate, Z).

The case of a constant relative risk aversion utility function

To reach clear-cut conclusions, I henceforth assume a constant relative risk aversion utility function :

$$v(R_t) = \begin{cases} \frac{R_t^\lambda}{\lambda} & \lambda \leq 1, \lambda \neq 0 \\ \ln(R_t) & \lambda = 0. \end{cases} \tag{25}$$

Moreover, I assume that the replacement ratio is constant and exogenous in each period t ($\frac{Z_t}{w_t} = z, z \in]0,1[$)[5] and that the basic income is proportional to the level of the unemployment benefit ($B_t = \xi Z_t$, $\xi \geq 0$). Let

$$\mathcal{I}(\xi) = \begin{cases} \xi & \text{if } \xi \geq 1 \text{ (the full basic income case)} \\ 1 & \text{if } \xi < 1 \text{ (the partial basic income case).} \end{cases} \tag{26}$$

[5]This assumption is supported by Figure 2.2 in OECD (1996).

With these assumptions and notation, the *wage-setting equation* (23) becomes :

$$\frac{1+\xi z}{\mu\lambda}\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right) \; + \; \beta(1-q)\left(\frac{w_{t+1}}{w_t}\right)^{\lambda} = \frac{q+(1-q)u_{t-1}}{u_t} \tag{27}$$

if $\lambda \neq 0$. This curve will be called the 'WS' curve. If $\lambda = 0$, the *wage-setting equation* (23) becomes a relationship between the current unemployment rate and the last period's one :

$$\frac{1+\xi z}{\mu}\ln\left(\frac{1+\xi z}{z\mathcal{I}(\xi)}\right) \; + \; \beta(1-q) = \frac{q+(1-q)u_{t-1}}{u_t}. \tag{28}$$

The budget constraint of the State

The budget of the State is assumed to be balanced in each period. This budget constraint is different whether the partial or the full basic income applies. It is also influenced by the degree of eligibility of the inactive population. Various assumptions could be made here. However, the theoretical analysis can stay fairly general. Let ν be the sum of the n firm owners and the size M of the (eligible) inactive population divided by the workforce ($\nu = \frac{n+M}{N}$). Ignoring administrative costs, a balanced budget has the following form :

$$w_t\tau_t(1-u_t) = \begin{cases} Z_t u_t + B_t(1-u_t+\nu) & \text{if a partial basic income applies} \\ B_t(1+\nu) & \text{if a full basic income applies} \end{cases} \tag{29}$$

or equivalently,

$$\tau_t = \begin{cases} z\left(\frac{u_t}{1-u_t} + \xi(1+\frac{\nu}{1-u_t})\right) & \text{if a partial basic income applies} \\ \xi z\frac{1+\nu}{1-u_t} & \text{if a full basic income applies.} \end{cases} \tag{30}$$

It can easily be checked that the tax wedge increases with the unemployment rate.

In the initial period ($t = 0$), the definition of the tax wedge can be combined with the aggregate labor demand curve. Starting from equation (10), this curve can be written as a static upward-sloping relationship between the unemployment rate and the real wage cost (often called the 'PS' curve) :

$$N(1-u_0) = \frac{\mathcal{K}_0}{A}\left(\frac{w_0(1+\tau_0)}{\alpha A}\right)^{\frac{1}{\alpha-1}}, \tag{31}$$

where $\mathcal{K}_0$ designates the aggregate capital stock ($\mathcal{K}_0 = nK_0$). The balanced budget equations can be used to eliminate τ_0 from the labor demand curve (31). This yields the following relationships between the net real wage rate w_0 and the unemployment rate u_0 :

$$w_0 = \begin{cases} \phi_t \frac{(1-u_0)^\alpha}{(1-u_0)+zu_0+\xi z(1-u_0+\nu)} & \text{if a partial basic income applies,} \\ \phi_t \frac{(1-u_0)^\alpha}{(1-u_0)+\xi z(1+\nu)} & \text{if a full basic income applies,} \\ \text{with } \phi_0 = \alpha A \left(\frac{AN}{\mathcal{K}_0}\right)^{\alpha-1}, & \phi_0 > 0. \end{cases} \tag{32}$$

This is nothing else than the so-called 'price-setting curve' PS extended to take the budget constraint of the State into account. Hence, this curve will be called the 'extended PS' curve. It can be checked that the curves (32) cut the vertical axis at $w_0 = \frac{\phi_0}{1+\xi z(1+\nu)} > 0$ and the horizontal one at $u_0 = 1$. The slope of these curves can be positive for small values of u_0 (if a reduction in employment increases marginal productivity sufficiently) but it necessarily becomes negative as u_0 increases. In the partial basic income case, it can be seen that the curve (32) is monotonically decreasing for plausible values of the parameters.

For $t \geq 1$, the definition of the tax wedge can be combined with the real input prices frontier (12) in order to generate a relationship between the net wage rate and the unemployment rate. After some manipulation, the 'extended PS curve' writes now :

$$w_t = \begin{cases} \frac{C(1-u_t)}{1+\xi z(1+\nu)-(1+(\xi-1)z)u_t} & \text{if a partial basic income applies} \\ \frac{C(1-u_t)}{1+\xi z(1+\nu)-u_t} & \text{if a full basic income applies.} \end{cases} \tag{33}$$

It can easily be checked that in both cases this curve is downward-sloping and concave. Moreover, it cuts the vertical axis at $w_t = \frac{C}{1+\xi z(1+\nu)} > 0$ and the horizontal one at $u_t = 1$.

The symmetric equilibrium

This economy is fully characterized by the wage-setting equation (27) or (28) and the 'extended PS curve' (32) if $t = 0$ and (33) for $t \geq 1$. The emphasis will be put on the latter case but let us first briefly look at the initial period. If $\lambda \neq 0$, conditional on w_1 and u_{-1}, $\frac{\partial w_0}{\partial u_0}$ has the same sign as λ (see (27)). Therefore, according to this sign and the shape of the 'extended PS curve' (32), the initial period equilibrium can be unique or not, conditional on w_1 and u_{-1}. If $\lambda = 0$, equation (28) directly determines u_0 conditional on u_{-1}. The equilibrium in $t = 0$ is then unique.

For $t \geq 1$, if $\lambda = 0$, the dynamics of the economy is quite simple. Equation (28) immediately provides a relationship between the current unemployment rate and the previous one. Appendix 1 proves the stability of this first-order dynamic equation. In the more general case ($\lambda \neq 0$), the ratio $\frac{w_{t+1}}{w_t}$ in equation (27) can be derived from equation (33) (evaluated at times t and $t+1$). After some manipulation, equation (27) becomes then a second-order scalar nonlinear difference equation where the current unemployment rate u_t is a function of the lagged unemployment level u_{t-1} and the future one u_{t+1}. Appendix 1 shows that this dynamic system is stable and that the equilibrium is a saddle point. Let us now concentrate on the steady state. In such a state, the unemployment rate stays constant. So does the tax wedge (see (30)) and the net real wage rate (see (33)). Therefore, the wage-setting equations define the equilibrium unemployment rate u^*. From (27) and (28), it can be shown that

$$u^* = \begin{cases} \frac{q}{\frac{1+\xi z}{\mu\lambda}\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right)-(1-\beta)(1-q)} & \text{if } \lambda \neq 0 \\ \frac{q}{\frac{1+\xi z}{\mu}\ln\left(\frac{1+\xi z}{z\mathcal{I}(\xi)}\right)-(1-\beta)(1-q)} & \text{if } \lambda = 0 \end{cases} \tag{34}$$

Section 4 will analyze the properties of the equilibrium unemployment rate. Knowing u^*, the steady state tax wedge and net real wage rate are easily computed from, respectively, (30) and (33). Figure 1 illustrates this solution. The equilibrium (w^*, u^*) is at the intersection of a vertical line 'WS' (34) and a downward-sloping 'extended PS' curve (33).

To sum up, in this model where the interest rate is exogenous, the real wage cost is exogenously fixed along the the real input prices frontier. The union-firm bargaining over the net real wage eventually plays the role of defining the equilibrium unemployment rate and, hence, the equilibrium aggregate output level. For this unemployment rate, the net real wage is read along the 'extended PS curve'. This implies that the net real wage rate accommodates any change in the tax rate induced by fluctuations in unemployment.

INSERT FIGURE 1 APPROXIMATELY HERE.

4 Comparative static analysis

This section analyzes the effect of the exogenous parameters on the equilibrium unemployment rate. It is essentially concerned with the impact of the basic income schemes on the equilibrium unemployment rate, net wage and tax wedge. To avoid clutter, no superscript is added to indicate that the endogenous variables are at their equilibrium level. This section only deals with the case of an iso-elastic instantaneous utility function (25). Due to space limitations, it focusses on the case $\lambda \neq 0$.

The equilibrium unemployment rate is strictly bounded between zero and one (see appendix 2). Moreover, as usual in this type of model, the levels of wages and unemployment depend crucially on unions' bargaining power and preferences and on the wage elasticities of profit and labor demand functions on the one hand and on the replacement ratio on the other hand (see, e.g., Layard, Nickell and Jackman, 1991). The following result summarizes some standard properties (the proof is left to appendix 2).

Result 1 *The equilibrium unemployment rate is an increasing function of the interest rate r, the separation rate q, the mark-up μ and the replacement ratio z. On the contrary, the equilibrium unemployment rate is lower the more relative risk averse workers are.*

Let us now turn to the main results of this paper.

Result 2 *Compared to the case with an unemployment insurance system but without basic income, the equilibrium unemployment rate is always lower if a partial basic income scheme is implemented. The same holds if the unemployment insurance system is abolished and replaced by a full basic income scheme if the ratio between the basic income and the unemployment benefit, ξ ($\xi > 1$), is lower than $1 + \frac{z}{1-z}$, where z is the replacement ratio.*

Let u_z denote the equilibrium unemployment rate when there is an unemployment insurance system (with a replacement ratio z) and no basic income scheme ($\xi = 0$) :

$$u_z = \frac{q}{\frac{1}{\mu\lambda}\left(1 - z^{\lambda}\right) - (1-\beta)(1-q)}.$$

The equilibrium unemployment rate u_z is higher than u defined in (34) if

$$\frac{1 - z^{\lambda}}{\lambda} < \frac{1 + \xi z}{\lambda}\left(1 - \left[\frac{z\mathcal{I}(\xi)}{1 + \xi z}\right]^{\lambda}\right).$$

This inequality is satisfied if the following condition holds :

$$\frac{1 - z^{\lambda}}{\lambda} < \frac{1}{\lambda}\left(1 - \left[\frac{z\mathcal{I}(\xi)}{1 + \xi z}\right]^{\lambda}\right).$$

The latter condition is verified if $1 + \xi z > \mathcal{I}(\xi)$. The last inequality is always satisfied in the case of a partial basic income $(0 < \xi < 1, \mathcal{I}(\xi) = 1)$. With a full basic income, the same conclusion holds if ξ is not too high $(1 \leq \xi < 1 + \frac{z}{1-z})$.

■

To understand more intuitively result 2, remember that expression

$$\frac{1 + \xi z}{\mu\lambda}\left(1 - \left[\frac{z\mathcal{I}(\xi)}{1 + \xi z}\right]^{\lambda}\right) \tag{35}$$

in (34) is a particular case of the ratio $\frac{v(w+B)-v_u}{\mu w v'(w+B)}$, which was first introduced in the static model of section 2. As can be seen from (35), the basic income affects the equilibrium unemployment rate through two channels : the term $1 + \xi z$ and the *generalized* replacement ratio $\frac{z\mathcal{I}(\xi)}{1+\xi z}$. The literature about wage-setting in the presence of a non linear tax system (Lockwood and Manning (1993)) clarifies the mechanism behind the first channel. According to this literature, an increase in the progressivity of taxation, loosely speaking, acts as an incentive for wage moderation. Similarly, *CZ* have shown that an increase in progressivity decreases the equilibrium unemployment rate. This result applies here, too. Whereas the tax wedge is constant, the net real income of the workers, $R_t = w_t + B_t$, can be rewritten as $w_t - T(w_t, B_t)$, where T is the amount of taxes incident upon the employee net of all benefits (here, $T(w_t, B_t) = -B_t$). Hence, the so-called coefficient of residual income progression (i.e. the elasticity of R_t with respect to w_t) is here equal to $\left(1+\frac{B_t}{w_t}\right)^{-1}$. Since this expression is lower than 1, taxes net of the basic income can be said to be progressive[6]. Moreover, the higher B, the more progressive the system is. To sum up, the basic income has a favorable effect on the equilibrium unemployment level partly because it introduces some progressivity in taxation.

The following proposition deals with the *slope* of the relationship between the steady state unemployment rate and the magnitude of the basic income. From (34), it is clear that the sign of $\frac{\partial u}{\partial \xi}$ is given by the sign of the partial derivative of (35). Therefore, the next result is closely connected to those presented in the partial equilibrium setting of section 2.

Result 3 *The equilibrium unemployment rate and net wage rate decrease with the level of the partial basic income. The equilibrium unemployment rate increases with (resp., is independent of) the level of the full basic income if workers are risk averse (resp., risk neutral). The equilibrium net wage rate decreases with the level of the full basic income.*

One has simply to carry out the first-order partial derivative of (34) with respect to ξ under both assumptions about $\mathcal{I}(\xi)$. In the partial basic income case ($0 < \xi < 1$), this calculation yields

$$\frac{\partial u}{\partial \xi} = -\frac{qz}{D^2\mu\lambda}\left[1-(1-\lambda)\left(\frac{z}{1+\xi z}\right)^{\lambda}\right] \; < \; 0,$$

where D is the denominator of (34). The expression between brackets is strictly positive if $\lambda = 1$. The same is true if $0 < \lambda < 1$ since $0 < \left(\frac{z}{1+\xi z}\right)^{\lambda} < 1$. When λ is negative, the expression between brackets is negative. Therefore the whole expression is negative, too. In the full basic income case ($\xi \geq 1$), it can easily be checked that

$$\frac{\partial u}{\partial \xi} = -\frac{qz}{D^2\mu\lambda}\left[1-(1+\frac{\lambda}{\xi z})\left(\frac{\xi z}{1+\xi z}\right)^{\lambda}\right] \; \geq \; 0. \tag{36}$$

[6] The hypothesis according to which the levels of the unemployment benefit and the basic income are independent of the negotiated wage rate in a *particular* firm plays here a crucial role.

Here the term between brackets is zero if $\lambda = 1$. On the contrary when $0 < \lambda < 1$, this expression is negative. The latter conclusion can be checked by numerical simulation for $0 < z < 1$, $0 < \lambda < 1$ and for plausible values of ξ (say, $1 \leq \xi \leq 2$). For sufficiently negative values of λ, the term between brackets is positive. Hence, $\frac{\partial u}{\partial \xi}$ is positive, too. Numerical simulations shows that the same is true when λ is negative but close to zero. The unambiguous sign of expression (36) is not in accordance with the equivalent expression in section 2 (see (8)). There, for sufficiently risk averse workers, the full basic income had a favorable effect on employment. It can be checked that the sign of (36) can become negative with a more general utility function than the isoelastic one. Yet, whether an equilibrium exists in such a case remains an open question.
As far as the net wage rate is concerned, it can easily be checked that $\frac{\partial w_t}{\partial \xi} \leq 0$ along the 'extended PS' curve (see (33)). Therefore, the equilibrium net wage rate decreases with ξ. ■

Figure 2 illustrates the effect of an increase in ξ on the equilibrium unemployment and net wage rates. The bold face curves represent the 'WS' and 'extended PS' curves after an increase in ξ. As a corollary, the negative sign of $\frac{\partial w_t}{\partial \xi}$ at the equilibrium implies that the equilibrium tax wedge increases with ξ (remember that the wage cost is actually exogenous and independent of ξ).

INSERT FIGURE 2 APPROXIMATELY HERE.

The previous results can be rephrased in a clear-cut qualitative message. Three sentences summarize this message. (1) If, for whatever reason, the unions' power and preferences and the replacement ratio are given, the introduction of a *partial* basic income lowers the equilibrium unemployment rate. (2) Moreover, if this policy is implemented, the highest possible *partial* basic income is recommended if the reduction of the unemployment rate is the unique government's goal. (3) However, this policy implies an increase in the tax wedge.

No discussion of the pros and cons of a basic income proposal is complete without at least a rough estimation of its main effects. As a first step towards this requirement, the next section presents a numerical example based on plausible values of the parameters. In addition, this section will illustrate that the introduction of a partial basic income can be a Pareto improvement.

5 A numerical example

In this section, I calibrate the model using some plausible assumptions for the parameters and focus on the steady state values of unemployment and taxation. This information is insufficient to advocate or not the reform under concern. Therefore, I will close this section with a brief welfare analysis.

Since wages are by assumption determined for one period, each period is assumed to last a year. The example is built upon the following values for the parameters : $\alpha = 0.7, \gamma = 0.6, \psi = 0$, hence $\mu = 0.64, \lambda = -1, \beta = 0.95, \nu = 0.1, q = 0.2$ and $z = 0.4$. In other words, the bargaining is modelled as the maximization of an asymmetric Nash product and the unions do not value the level of employment[7]. The assumption $\nu = 0.1$ means that, on average in the EU, about one-quarter of the inactive population aged 18-64 would be eligible for a basic income. Considering only the population aged 18-64 stems from the focus of this paper on the unemployment insurance mechanism[8]. Moreover, it is assumed that some *participation* criteria restrict eligibility. In this context, the assumption of an eligibility rate of 25% is simply an example. However, remember that ν influences only the tax wedge and that the magnitude of the latter has no effect on the unemployment rate under the assumptions behind (34). The value of the separation rate q is in accordance with the results of Burda and Wyplosz (1994). $z = 0.4$ is an hypothesis that could be

[7]The assumption $\psi = 0$ is in accordance with the so-called seniority model. Moreover, sufficiently close to the steady state, each union member is certain to keep his job since new hirings should compensate the number of quits that occurred at the end of the previous period. Hence, the assumption $\psi = 0$ is plausible in the neighborhood of the steady state.

[8]Other components of the Welfare State such as state pensions and family benefits have been left aside.

supported by the results provided in OECD (1996)[9].

Figure 3 illustrates results 2 and 3. The equilibrium unemployment rate is highest in the absence of a basic income, it decreases as a function of ξ, reaches its minimum when the basic income equals the level of the unemployment benefit and then starts increasing. The relative decrease in the equilibrium unemployment rate is large (from about 9% when $\xi = 0$ to less than 4% when $\xi = 1$). It should be added that unreported numerical simulations indicate that the unemployment rate varies monotonically along the dynamic path between two equilibria.

Figure 3 also shows that the tax wedge τ is nearly proportional to ξ. The wedge is about 15% when ξ equals 0.3 (i.e. a basic income-net wage ratio of 12%). In addition to the tax wedge, it is interesting to have a look at the average tax rate (in the case of an occupied worker), θ, defined by the equality $B + w = (1-\theta)w(1+\tau)$ with $B = \xi z w$. Hence, $\theta = 1 - \frac{1+\xi z}{1+\tau}$. As Figure 3 shows, θ is a slightly increasing function of ξ. It amounts to 3.5% when ξ equals 0.3. These results raise the issue of the trade-off between unemployment and the tax rate. A traditional argument against any basic income scheme is that the large increase in the tax wedge implied by this policy outweighs its advantages. This paper only deals with one of them, namely the reduction in the unemployment rate. There is therefore no claim that this analysis definitely clarifies this issue.

INSERT FIGURE 3 APPROXIMATELY HERE.

Nevertheless, it is interesting to notice that the introduction of a partial basic income is a Pareto improvement in this example if the ratio ξ is sufficiently low ($\xi \leq 0.3$). There is no claim that this property applies for a large range of values of the parameters. Yet, the fact that this reform can be a Pareto improvement is as such a valuable result.

This economy is ex post made of four types of agents (firm-owners, the inactive population, the occupied workers and the unemployed). It is easily seen that the steady state

[9] It turns out to be a bit higher than the Belgian aggregate net replacement ratio at the beginning of the nineties (see Dor, Van der Linden and Lopez-Novella, 1997).

value of profits is zero. Hence, if firm owners are eligible to the basic income, their intertemporal utility becomes positive thanks to this grant (otherwise it remains unchanged). The same is true for those in the inactive population. Things are less obvious for the two other categories. From (15), (20) and (21), it can be verified that the steady state intertemporal utilities of respectively an occupied worker and an unemployed are given by :

$$\begin{aligned}
V_e &= \frac{v(w+B) - \beta q \mu w v'(w+B)}{1-\beta} = \frac{C^\lambda}{\lambda(1-\beta)}\left(\frac{1+\xi z}{1+\tau}\right)^\lambda \left(1 - \frac{\beta q \mu \lambda}{1+\xi z}\right) \\
V_u &= V_e - \frac{\mu w v'(w+B)}{1-a} = \frac{C^\lambda\,(1+\xi z)^{\lambda-1}}{(1+\tau)^\lambda}\left[\frac{1+\xi z - \beta q \mu \lambda}{\lambda(1-\beta)} - \mu\left(\frac{q}{u} + 1 - q\right)\right].
\end{aligned}$$

Neglecting the scaling factor C^λ, Figure 4 illustrates these formula. This figure assumes the values of the parameters indicated at the beginning of this section. It shows that both V_e and V_u increase as long as ξ is small. For higher values of ξ, V_e still increases but at a slowing rate. V_e reaches a maximum for $\xi = 1$. As far as V_u is concerned, it reaches a local maximum about $\xi = 0.3$ and it starts increasing again when a full basic income replaces the partial scheme (the immediate effect on income outweighs the reduction in the hiring rate).

INSERT FIGURE 4 APPROXIMATELY HERE.

6 Conclusion

The economics of basic income schemes has often been reduced to an arithmetic exercise or it has focussed on labor supply in a competitive setting. This paper departs from these viewpoints and analyzes the wage and employment effects of such a scheme in static and dynamic models of collective bargaining. In European countries at least, the latter assumption is much more plausible than the competitive one.

I have considered two variants of the basic income proposal. In the first case, called the full basic income scheme, the unconditional grant replaces all social insurance and social assistance benefits for the population of working age. In the second case, called a partial basic income, the benefits that are lower than the basic income disappear while the others are reduced by the amount of the partial basic income. The analysis has assumed that an unemployment insurance system initially exists and it has ignored the other components of the social security and assistance systems. The level of the unemployment benefits was taken as given. The analysis has concentrated on reforms that either leave the net income of the unemployed unchanged (the partial scheme) or improves it (the full scheme). Yet, the model developed in this paper is probably also a useful framework for the proponents of the view that a basic income should both replace all social security benefits and be lower than the latter.

To ease the understanding of the effect of a basic income on wage bargaining, this paper has first developed a partial equilibrium and static framework. This simple model has afterwards been embedded in a dynamic general equilibrium setting. The analysis ends with a number of strong predictions. First, compared to the case with an unemployment insurance system but without basic income, the equilibrium unemployment rate is always lower if a partial basic income scheme is implemented. The same holds if the unemployment insurance system is abolished and replaced by a full basic income scheme provided that the ratio between the basic income and the unemployment benefit is sufficiently small. Second, the equilibrium unemployment rate is a decreasing function of the level of the partial basic income. Third, the equilibrium unemployment rate is an increasing function of the level of the full basic income if workers are risk averse. Fourth, the equilibrium tax wedge increases with the level of the basic income while the net wage rate decreases. Finally, introducing a sufficiently small partial basic income can be a Pareto improvement.

This paper has abstracted from many important features of a basic income proposal. First, agents were assumed to be identical. It is true that *ex post* some of them were unemployed and some others not. But many income redistribution issues have been left aside. Nor has this paper dealt with the effect of a basic income on poverty, the degree of dependency on means-tested benefits or the pattern of power and dependency within families. Second, the instantaneous utility function was assumed to be a function of income only. Third, economic agents took their decision in a deterministic and full information setting. Consequently, any conflict between employers and employees about the level of effort of the latter was ruled out. Finally, this paper has neglected administrative costs and oversimplified institutional features (e.g. those of the unemployment benefit system). These shortcomings and possibly others suggest interesting ways to pursue this line of analysis one step further.

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Appendix 1

This appendix deals with the dynamic properties of the equilibrium, with a focus on unemployment. It is organized as follows. First, it briefly deals with the case $\lambda = 0$. Secondly, under the assumption $\lambda \neq 0$, it shows that the dynamic behavior of this economy can be represented by a second-order scalar nonlinear difference equation where the current unemployment rate u_t is a function of the lagged unemployment rate u_{t-1} and the future one u_{t+1}. Thirdly, it analyzes the properties of this difference equation.

When the utility function $v(R_t) = \ln(R_t)$, i.e. $\lambda = 0$

The dynamics of the economy is here quite simple. Equation (28) immediately provides a relationship between the current unemployment rate and the previous one. This relationship can be rewritten as

$$u_t = \rho u_{t-1} + \rho \frac{q}{1-q}, \ \rho = \frac{1-q}{\frac{1+\xi z}{\mu} \ln\left(\frac{1+\xi z}{z\mathcal{I}(\xi)}\right) + \beta(1-q)}.$$

This first-order equation is stable if $\rho < 1$, i.e. if

$$\frac{1+\xi z}{\mu} \ln\left(\frac{1+\xi z}{z\mathcal{I}(\xi)}\right) - (1-\beta)(1-q) > 0.$$

This condition imposes that the equilibrium unemployment rate be positive (see (34)). This is the case for plausible values of the parameters.

The dynamic system when $\lambda \neq 0$

Here, the utility function is $v(R_t) = \frac{R_t^\lambda}{\lambda}$. For $t \geq 1$, the ratio $\frac{w_{t+1}}{w_t}$ in equation (27) can be derived from equation (33). It can be checked that for $t \geq 1$ the unemployment rate fluctuates according to the following second-order equations :

$$\theta_1 \left(\frac{\theta_2 - \theta_3 u_t}{\theta_2 - \theta_3 u_{t+1}} \frac{1 - u_{t+1}}{1 - u_t} \right)^\lambda - \frac{q + (1-q) u_{t-1}}{u_t} + \theta_4 = 0, \tag{37}$$

with a partial basic income scheme and

$$\theta_1 \left(\frac{\theta_2 - u_t}{\theta_2 - u_{t+1}} \frac{1 - u_{t+1}}{1 - u_t} \right)^\lambda - \frac{q + (1-q) u_{t-1}}{u_t} + \theta_4 = 0, \tag{38}$$

with the full basic income scheme. In these expressions,

$$\begin{aligned}
\theta_1 &= \beta(1-q), \ \theta_1 \in]0,1[, \\
\theta_2 &= 1 + \xi z(1+\nu), \ \theta_2 > 1, \\
\theta_3 &= 1 + (\xi - 1) z, \ \theta_3 \in]0,1[, \\
\theta_4 &= \frac{1 + \xi z}{\mu \lambda} \left(1 - \left[\frac{z \mathcal{I}(\xi)}{1 + \xi z} \right]^\lambda \right), \ \theta_4 > 0.
\end{aligned}$$

Let $F^j(u_{t-1}, u_t, u_{t+1}) = 0$ ($j = 1, 2$) denote respectively the second-order scalar nonlinear difference equations (37) and (38). Let us linearize $F^j(u_{t-1}, u_t, u_{t+1}) = 0$ around the steady state u^{j*} (defined in (34)). Remember that the steady state is defined differently whether $\xi < 1$ (i.e. $j = 1$) or $\xi \geq 1$ (i.e. $j = 2$). The linearized difference equation can be converted to an equivalent first-order planar map :

$$\begin{pmatrix} u_{t+1} - u^{j*} \\ u_t - u^{j*} \end{pmatrix} = \mathcal{A}^j \begin{pmatrix} u_t - u^{j*} \\ u_{t-1} - u^{j*} \end{pmatrix} \tag{39}$$

In this expression, $\mathcal{A}^j$ is the 2×2 matrix

$$\begin{pmatrix} -\frac{F_2^j(u^{j*})}{F_3^j(u^{j*})} & -\frac{F_1^j(u^{j*})}{F_3^j(u^{j*})} \\ 1 & 0 \end{pmatrix} \tag{40}$$

where F_k^j denotes the first-order partial derivative of F^j with respect to its kth argument.

The properties of dynamic system when $\lambda \neq 0$

In (40), the first-order partial derivatives of F^j can be written as

$$\begin{aligned}
F_1^j(u^{j*}) &= -\frac{1-q}{u^{j*}}, \\
F_2^j(u^{j*}) &= \zeta^j + \frac{q + (1-q) u^{j*}}{(u^{j*})^2}, \\
F_3^j(u^{j*}) &= -\zeta^j,
\end{aligned}$$

where

$$\begin{aligned}
\zeta^1 &= \lambda\theta_1 \frac{\theta_2 - \theta_3}{(\theta_2 - \theta_3 u^{1*})(1 - u^{1*})}, \\
\zeta^2 &= \lambda\theta_1 \frac{\theta_2 - 1}{(\theta_2 - u^{2*})(1 - u^{2*})}.
\end{aligned}$$

For $j = 1, 2$, since $\theta_2 > 1$ and $\theta_3 \in]0, 1[$, it can be checked that ζ^j has the same sign as λ.

The characteristic polynomial is $P^j(\omega) = \omega^2 - (tr\mathcal{A}^j)\omega + (det\mathcal{A}^j)$ where

$$\begin{aligned}
(tr\mathcal{A}^j) &= -\frac{F_2^j(u^{j*})}{F_3^j(u^{j*})} = 1 + \frac{q + (1-q)u^{j*}}{\zeta^j (u^{j*})^2}, \\
(det\mathcal{A}^j) &= \frac{F_1^j(u^{j*})}{F_3^j(u^{j*})} = \frac{1-q}{\zeta^j u^{j*}}, \text{ with } sgn(det\mathcal{A}^j) = sgn(\zeta^j).
\end{aligned}$$

With one predetermined variable, the saddle point property is required in order to have a unique nonexploding solution. This property is guaranteed if $(tr\mathcal{A}^j)^2 - 4(det\mathcal{A}^j) > 0$ and if $[P^j(1) < 0$ and $P^j(-1) > 0]$ or $[P^j(1) > 0$ and $P^j(-1) < 0]$. Let us check these conditions. First, $(tr\mathcal{A}^j)^2 - 4(det\mathcal{A}^j)$ is always positive if ζ^j is negative. When $\zeta^j > 0$, $(tr\mathcal{A}^j)^2 - 4(det\mathcal{A}^j)$ is positive if

$$\left(1 + \frac{q}{\zeta^j (u^{j*})^2}\right)^2 + \frac{1-q}{\zeta^j u^{j*}} \left(\frac{2q}{\zeta^j (u^{j*})^2} + \frac{1-q}{\zeta^j u^{j*}} - 2\right) > 0. \tag{41}$$

There is no proof that this condition is always satisfied. Yet, numerical simulations show that it is verified if $0.01 < q < 0.4$ and $0 \leq u^{j*} \leq 1$. Since this sub-space covers the range of plausible values, condition (41) should be considered as fulfilled.

Let us now turn to the second set of conditions. Carrying out the calculation yields

$$\begin{aligned}
P^j(1) &= -\frac{q}{\zeta^j (u^{j*})^2}, \text{ with } sgn[P^j(1)] = -sgn[\zeta^j], \\
P^j(-1) &= 1 + tr\mathcal{A}^j + det\mathcal{A}^j = 2 + \frac{q + 2(1-q)u^{j*}}{\zeta^j (u^{j*})^2}.
\end{aligned}$$

When $\lambda > 0$, it is easily seen that $P^j(1) < 0$ and $P^j(-1) > 0$. Put another way, the linearized dynamic system (39) has the saddle point property. When $\lambda < 0$, $P^j(1)$ is positive but $P^j(-1)$ is not necessarily negative. Yet, unreported numerical simulations show that u^{j*} sharply decreases for $\lambda < 0$, so that the sign of $P^j(-1)$ turns out to be negative for plausible values of the parameters. The saddle point property holds here, too.

Appendix 2

This appendix checks whether the equilibrium unemployment rate $u \in]0,1[$. Next, it proves result 1.

The equilibrium unemployment rate is bounded between zero and one
From (34), u is positive if

$$\frac{1+\xi z}{\mu\lambda}\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right) > (1-\beta)(1-q).$$

Whatever the sign of λ, the left-hand side is positive. Unreported numerical simulations show that this inequality is verified for plausible values of the parameters. u is lower than 1 if

$$\frac{1+\xi z}{\mu\lambda}\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right) > 1-\beta(1-q).$$

This inequality is satisfied according to unreported numerical simulations (again for plausible values of the parameters).

The proof of result 1
Let D be the denominator of (34), if $\lambda \neq 0$. Carrying out the appropriate first-order derivatives yields :

$$\begin{aligned}
\frac{\partial u}{\partial r} &= \frac{\partial u}{\partial \beta}\frac{\partial \beta}{\partial r} = \frac{q(1-q)}{D^2(1+r)^2} \ > \ 0, \\
\frac{\partial u}{\partial q} &= \frac{1}{D}\left(1-\frac{q(1-\beta)}{D}\right) \ > \ 0 \text{ since } u<1, \\
\frac{\partial u}{\partial \mu} &= \frac{q}{D^2\mu^2}\frac{1+\xi z}{\lambda}\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right) \ > \ 0, \\
\frac{\partial u}{\partial z} &= -\frac{q\xi}{\mu\lambda D^2}\left[1-\left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)^{\lambda}\left(1+\frac{\lambda}{\xi z}\right)\right] \ > \ 0.
\end{aligned}$$

The last property cannot be shown analytically. Yet, an unreported numerical analysis shows that it is verified for plausible values of the parameters. Finally, relative risk aversion equals $1-\lambda$ and

$$\frac{\partial u}{\partial \lambda} = \frac{q(1+\xi z)}{\mu\lambda D^2}\left(\frac{1}{\lambda}\left(1-\left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)^{\lambda}\right)+\ln\left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)\left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)^{\lambda}\right) \ > \ 0,$$

again on the basis of a numerical simulation. ■

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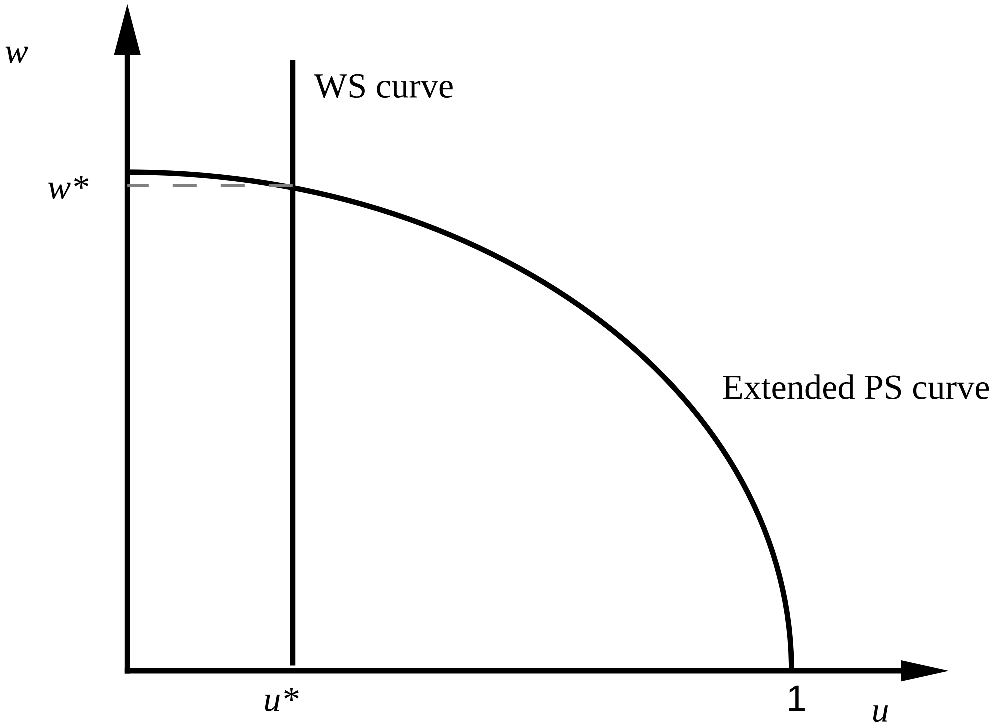
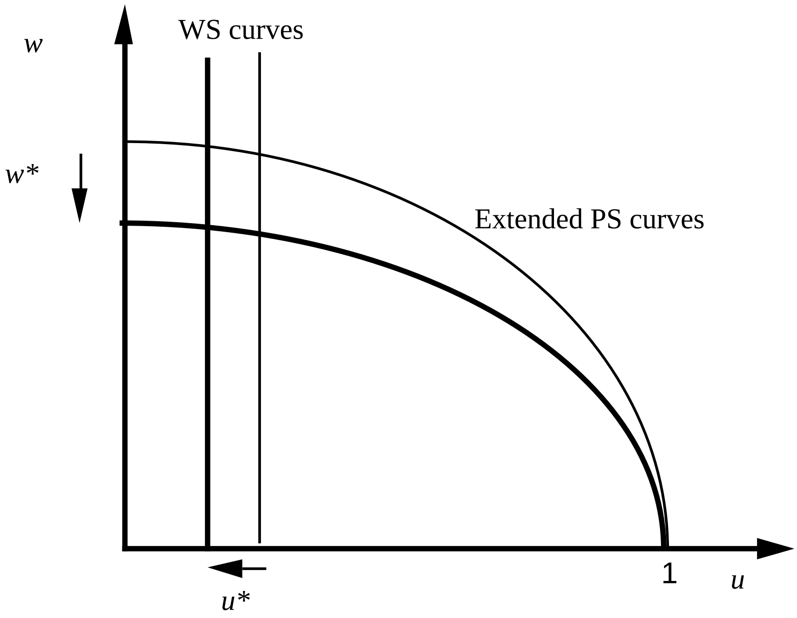
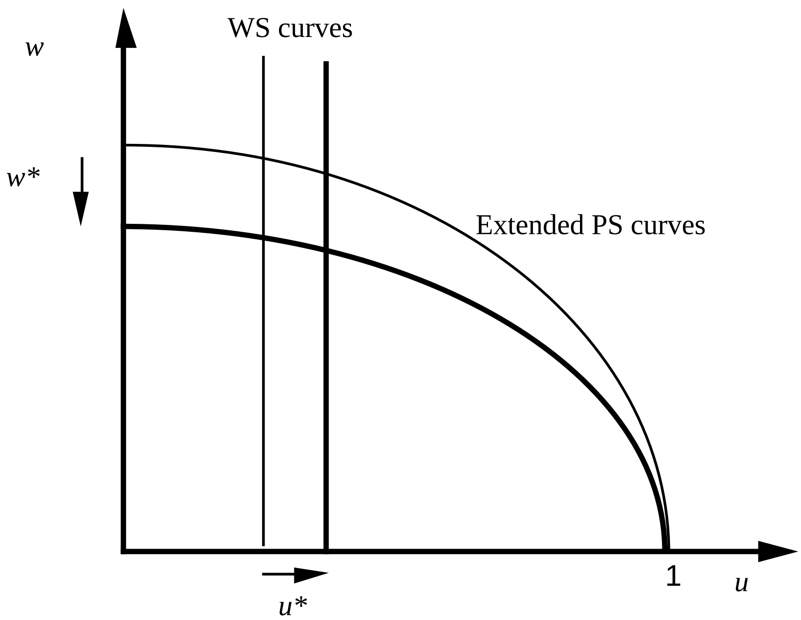


Figure 1: The equilibrium



The effect of an increase in the partial basic income.



The effect of an increase in the full basic income.

Figure 2: Comparative static analysis

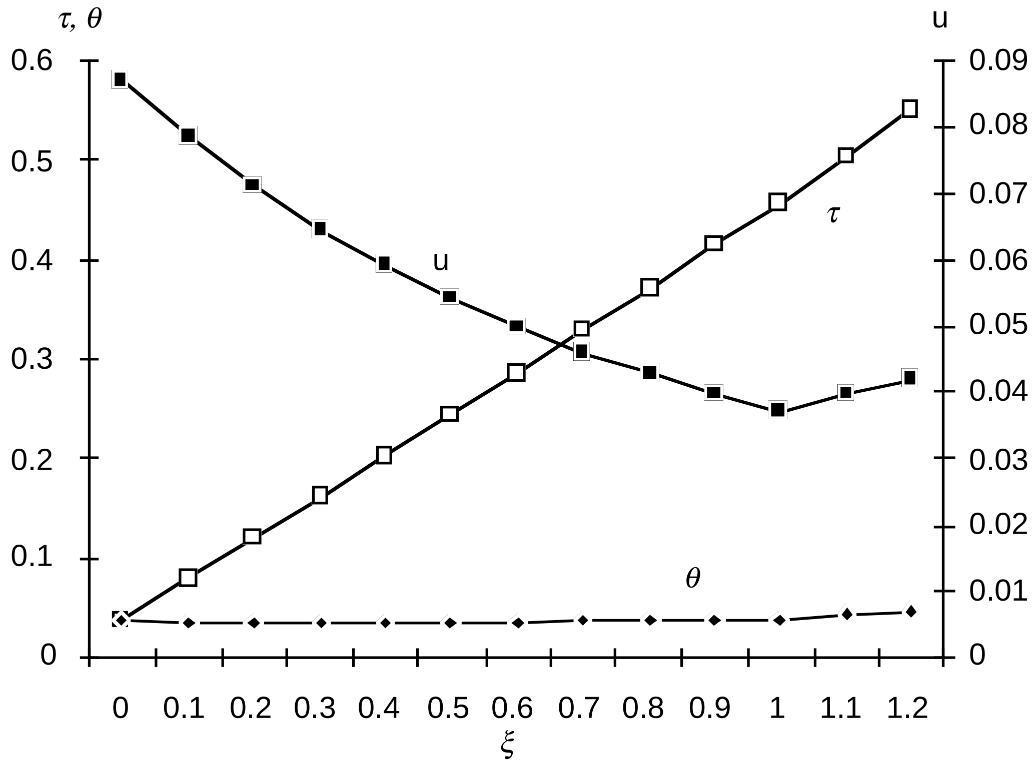


Figure 3: The equilibrium unemployment rate u, tax wedge τ and average tax rate θ

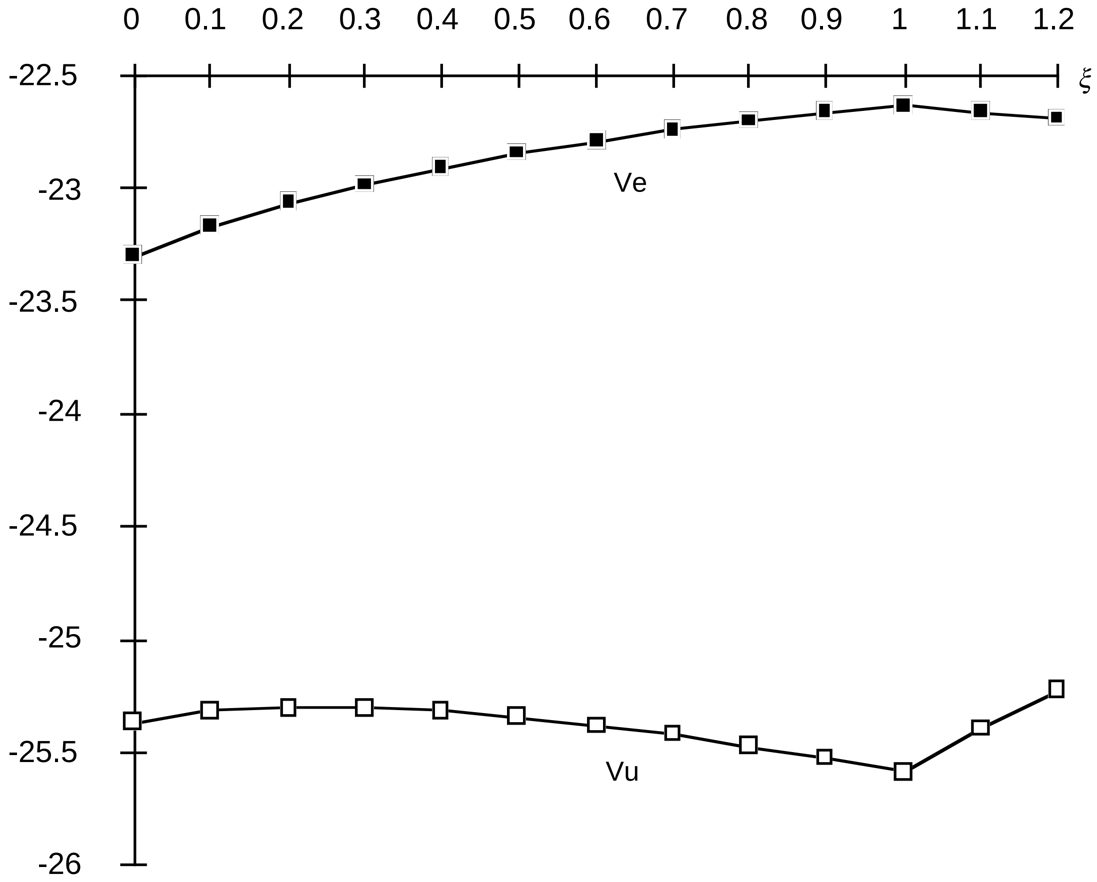


Figure 4: The equilibrium intertemporal utilities of an occupied worker V_e and an unemployed V_u